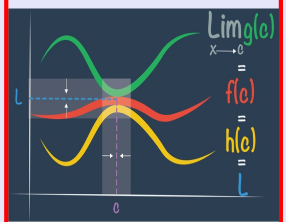
Math 261

Fall 2022

Lecture 20



Calculus I	**Name:** ______________________
Online Quiz 5	**Signature:** ______________________

No Work ⇔ No Points

Be Neat , Organized & Show Detailed Work

1. (5 points) Find $f'(x)$ for $f(x) = x^3 \csc x$.

$f'(x) = 3x^2 \csc x + x^3 \cdot (-\csc x \cot x)$

$= 3x^2 - x^3 \csc x \cot x$

1. ______

2. (5 points) Find $f'(x)$ for $f(x) = (x^2 - 2x)\cot x$.

$f'(x) = (2x-2) \cdot \cot x + (x^2-2x) \cdot (-\csc^2 x)$

$= (2x-2)\cot x - (x^2-2x) \cdot \csc^2 x$

2. ______

3. (5 points) Find $\frac{dy}{dx}$ for $xy = 4$.

$y = \frac{4}{x}$ $\quad y = 4x^{-1}$

$\frac{dy}{dx} = 4(-1) \cdot x^{-2} = \boxed{\frac{-4}{x^2}}$

3. $\frac{dy}{dx} = \frac{-4}{x^2}$

4. (5 points) Find $\frac{dx}{dy}$ for $y = \sqrt{x-2}$.

$y^2 = x - 2$

$x = y^2 + 2$

$\frac{dx}{dy} = 2y + 0 = \boxed{2y}$

4. $\frac{dx}{dy} = 2y$

Page 1 of 1 | Online Quiz 5 | Total Points: 20

Introduction to chain Rule:

$$\frac{d}{dx}\left[(f(x))^n\right] = n\,(f(x))^{n-1}\cdot\frac{d}{dx}\left[f(x)\right]$$

Find $\frac{dy}{dx}$ for $y = (x^3 - 4x)^5$

$$\frac{dy}{dx} = 5(x^3-4x)^{5-1}\cdot\frac{d}{dx}\left[x^3-4x\right]$$

$$= 5(x^3-4x)^4\cdot(3x^2-4)$$

Find $f'(x)$ for $f(x) = \left(\frac{x-2}{x+1}\right)^3$

$$f'(x) = 3\left(\frac{x-2}{x+1}\right)^2\cdot\frac{d}{dx}\left[\frac{x-2}{x+1}\right]$$

$$= 3\left(\frac{x-2}{x+1}\right)^2\cdot\frac{1(x+1)-(x-2)\cdot 1}{(x+1)^2}$$

$$= 3\cdot\frac{(x-2)^2}{(x+1)^2}\cdot\frac{\cancel{x}+1-\cancel{x}+2}{(x+1)^2} = \boxed{\frac{9(x-2)^2}{(x+1)^4}}$$

Find y' for $y = \cos^3 x$

$$y = \left[\cos x\right]^3$$

$$y' = 3\left[\cos x\right]^2\cdot -\sin x \qquad \boxed{y' = -3\cos^2 x\,\sin x}$$

Find $\frac{dy}{dx}$ for $y = \sqrt{\sin x + \tan x}$

$$y = (\sin x + \tan x)^{1/2}$$

$$y' = \frac{1}{2}(\sin x + \tan x)^{\frac{1}{2}-1}\cdot(\cos x + \sec^2 x)$$

$$y' = \frac{\cos x + \sec^2 x}{2(\sin x + \tan x)^{1/2}} \Rightarrow y' = \frac{\cos x + \sec^2 x}{2\sqrt{\sin x + \tan x}}$$

More general rule for chain rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Ex: $$\frac{d}{dx}\left[\sin(x^2)\right] = \cos(x^2) \cdot 2x$$
$$= 2x\cos(x^2)$$

$$\frac{d}{dx}\left[\tan(\sqrt{x})\right] = \sec^2(\sqrt{x}) \cdot \frac{d}{dx}[\sqrt{x}]$$

$$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{1/2}]$$
$$= \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$= \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$
$$= \boxed{\frac{\sec^2(\sqrt{x})}{2\sqrt{x}}}$$

Find $f'(x)$ for $f(x) = \left[\sec(\cos x^2)\right]^2$

$$f'(x) = 2\left(\sec(\cos x^2)\right)^1 \cdot \sec(\cos x^2) \cdot \tan(\cos x^2)$$
$$\cdot -\sin x^2 \cdot 2x$$

$$= -4x\left(\sec(\cos x^2)\right) \cdot \sec(\cos x^2) \cdot \tan(\cos x^2)$$
$$\cdot \sin x^2$$

$$\boxed{f'(x) = -4x\left(\sec(\cos x^2)\right)^2 \cdot \tan(\cos x^2) \cdot \sin x^2}$$

Find $f'(x)$ for $f(x) = \left(\frac{\sin x^2 + \cos x^2}{x^2}\right)^2$

$$f'(x) = 2\left(\frac{\sin x^2 + \cos x^2}{x^2}\right)^1 \cdot \frac{(\cos x^2 \cdot 2x - \sin x^2 \cdot 2x)x^2 - (\sin x^2 + \cos x^2)\cdot 2x}{(x^2)^2}$$

$$f'(x) = 2 \cdot \frac{\sin x^2 + \cos x^2}{x^2} \cdot \frac{2x^3(\cos x^2 - \sin x^2) - 2x(\sin x^2 + \cos x^2)}{x^4}$$

$$= \frac{2(\sin x^2 + \cos x^2)}{\cancel{x^2}} \cdot \frac{2\cancel{x}[x^2(\cos x^2 - \sin x^2) - (\sin x^2 + \cos x^2)]}{x^4}$$

$$= \frac{4(\sin x^2 + \cos x^2)[x^2(\cos x^2 - \sin x^2) - (\sin x^2 + \cos x^2)]}{x^5}$$

Find $\frac{dy}{dx}$ for $y = \underbrace{\sec^2 x^3 - \tan^2 x^3}$

$y = 1$

$y' = 0$

$$y' = 2\sec x^3 \cdot \sec x^3 \tan x^3 \cdot 3x^2$$

$$- 2\tan x^3 \cdot \sec^2 x^3 \cdot 3x^2$$

$$= 6x^2 \sec^2 x^3 \tan x^3 - 6x^2 \sec^2 x^3 \tan x^3$$

$$= \boxed{0}$$

$\sin^2 x + \cos^2 x = 1$

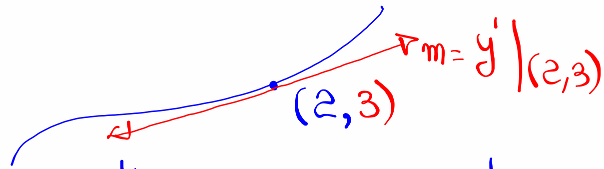
Divide by $\cos^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

Find eqn of the tan. line to the curve of $y = \sqrt{1+x^3}$ at the point with $x = 2$.



$y = (1 + x^3)^{1/2}$

$y' = \frac{1}{2}(1+x^3)^{-\frac{1}{2}} \cdot 3x^2$

$y' = \dfrac{3x^2}{2\sqrt{1+x^3}}$

$m = y'\big|_{(2,3)} = \dfrac{3 \cdot 2^2}{2\sqrt{1+2^3}} = \dfrac{3 \cdot 4}{2 \cdot 3} = 2$

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x-2)$

$\boxed{y = 2x - 1}$

$r(x) = f(g(h(x)))$

$h(1) = 2$, $g(2) = 3$

$h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$

Find $r'(1)$

$r'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

$r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1)$

$= f'(g(2)) \cdot g'(2) \cdot 4$

$= f'(3) \cdot 5 \cdot 4$

$= 6 \cdot 5 \cdot 4$

$= \boxed{120}$

Find x-value where $y = \sin 2x - 2\sin x$ has $\underbrace{\text{horizontal tan. line}}$ on $[0, 2\pi)$

$m = 0$

$y' = 0$

$y' = \cos 2x \cdot 2 - 2 \cdot \cos x$

$y' = 2[\cos 2x - \cos x]$

Solve $y' = 0$

$\cos 2x - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = \frac{-1}{2}$ $\qquad$ $\cos x = 1$

Make sure to solve for x in $[0, 2\pi)$